SYNOPSIS

Wave Propagation in Anisotropic & Inhomogeneous Structures

Ph D Thesis
Abir Chakraborty
S R No 115101334
Department of Aerospace Engineering
Indian Institute of Science
Bangalore - 560012, INDIA

With the advent of superior engineering materials, such as fibre reinforced composite (anisotropic) and Functionally Graded Materials (FGM) (inhomogeneous), there is a greater possibility of designing structures for harsh loading environments involving rapid build up of mechanical (impact) and thermal loading (atmospheric re-entry etc) The responses of these advanced materials are much more complex compared to the conventional structural materials, like metals and thus demand special attention

Impact is necessarily a short duration event that gives rise to dynamic stress propagation through the structure Impact loading has large frequency content and hence one has to perform wave propagation analysis in order to determine the transient responses Wave propagation analysis of orthotropic and inhomogeneous structures is very complex compared to metallic isotropic structures Hence, very few works have been reported so far in the literature

Although, Finite Element Method (FEM) still remains most versatile, accurate and generally applicable instrument for this kind of analysis, the method is computationally prohibitive for wave propagation analysis of large structures This is because for accurate prediction, the Finite Element (FE) size should be of the order of the wavelength, where the later is very small due to high frequency content of the load This constraint compels us to look for other non-FE methods

The need for cost-effective computation of wave propagation brings us to the realm of Spectral Finite Element Method (SFEM), which is tailor-made to study the wave propagation problems By virtue of its domain transfer formulation, it bypasses the large system size of FEM. Further, inverse problems like force and material property identification can be performed most conveniently and efficiently, compared to any other existing methods The SFEM has been used widely in the past for one and two dimensional isotropic materials and force identification problems More recently, it is applied for modeling one dimensional anisotropic material and Structural Health Monitoring (SHM) applications Still, its application to two-dimensional (2D) waveguides is not widespread due to difficulty in tracking various propagating modes The foremost requirement is a generalized approach based on the method of Partial Wave Techniques (PWT) The existing method cannot handle anisotropic and inhomogeneous multi-degrees of freedom structures, such as, beams, plates and shells, whereas, this new technique can be effectively used for the determination of various wave modes in these higher order waveguides In particular, successful implementation of this approach will result in the development of Spectral Finite Elements (SFEs) for modeling anisotropic and inhomogeneous one-dimensional (1D) structures, 2D layered media and 2D elementary and higher order plates and shells Further, these elements can be directly used in the solution of inverse problems in anisotropic and inhomogeneous structures and also in SHM applications

One of the fundamental requirement for the development of SFE is the availability of the exact solutions of the governing Partial Differential Equations (PDEs) However, they are available only for a few special cases For cases such as, coupled elastic and piezo-electric analysis and wave propagation in inhomogeneous structures, they are not available and we need to look for approximate solutions, which will retain the advantages of the SFE, e g , small system size and the ability to solve inverse problems One such approximate method which can be readily implemented in the SFE environment, is the Thin-Layer Method (TLM), which is very effective in modeling inhomogeneous and piezo-elastic material Alternatively, one can solve the static part of the governing PDEs exactly and use the solution in the FE formulation This generates what is called the Statically Exact Finite Elements (SEFEs) However, such elements can be formulated for one dimensional models only These alternate techniques enjoy the efficiency of SFEM and versatility of FEM, making them suitable for analysing wave propagation problems

Based on these considerations, three main goals are identified to be pursued in this thesis The first one is to develop 1D and 2D SFEs to analyze linear elastic wave motion in anisotropic and inhomogeneous media The second is the development of alternative methods like the TLM for modeling smart piezo-electric composite layer The third is the application of these developed elements towards solving inverse problems such as, material property estimation, impact force identification, detection of cracks or delamination in plate structure and some applications such as, active control of waves in plates and propagation of lamb waves in layered media.

The thesis consists of three parts. In the first part SEFEs and SFEs are developed to analyze wave propagation in anisotropic and inhomogeneous 1D waveguides. The material inhomogeneity is assumed both in the direction of wave propagation as well as in its normal direction. In the first case, the formulated SFEs are exact and equally applicable to anisotropic and inhomogeneous material, whereas, in the second case both exact and approximate SFEs are formulated. The approximate elements are based on the method of beam propagation technique, mostly used in optics. The developed SEFEs have exact static stiffness matrix and consistent mass matrix, where the effect of the first and second mass moments (rotary inertia) is included. Thus the elements become suitable for wave propagation analysis as the stiffness matrix is exact. All these elements are verified with results from 2D FE analysis. Further, in all the analyses, thermal effects are considered as an externally imposed field over the mechanical loading. From this exploration, we discover various aspects like the effect of inhomogeneity on propagating modes, its application in stress smoothing, effect of higher modes in the solution field, effect of the thermal field and wave propagation due to thermal burst loading.

In the second part, the 2D SFEs are developed to model anisotropic and inhomogeneou layered media, layered media with coupled thermo-elasticity and plate elements based on Kirchhoff's hypothesis and Mindlin plate theory. The TLM is employed to model anisotropic and inhomogeneous piezo-composite layered media and propagation of the elastic and electric modes is investigated. Similar to the case of 1D formulation, the 2D SFE for inhomogeneous material has both exact and approximate form and the performance of both anisotropic and inhomogeneous layer elements is verified with 2D FE analysis results. These developed elements are used to study the propagation of P and SV waves (QP and QSV for anisotropic layer), stress wave propagation and attenuation of stress waves, propagation of Rayleigh and Stoneley waves and the effect of temperature field on the mechanical response. Lamb wave modes are captured and their time domain representations are obtained using the SFEs. Effects of anisotropy and inhomogeneity on the Lamb wave modes are analysed in detail.

In the third part, several practical problems are addressed. These include solving inverse problems, e.g., force and material property estimation from the measured response, broad-band control of wave propagation in smart laminated piezo-composite plate and modeling and detection of crack in laminated plate structures. Impact loadings are reconstructed using the transfer function of the structure and frequency domain data of the truncated experimental response. The material properties are estimated by combined numerical-experimental technique, here named as the pulse propagation technique (PPT). In this method constrained nonlinear optimization is used along with experimentally measured response. While solving the inverse problems, FE solutions have been used as surrogate experimental data. Delamination is modeled by the previously developed spectral plate element and the scattering of waves in the presence of cracks is demonstrated. Finally, wave propagation control is studied using the novel Active Spectral Finite Element Method (ASFEM) for both open and closed-loop control.

This thesis collectively addresses all aspects pertaining to the wave propagation in anisotropic and inhomogeneous structures. In addition, the thesis dwells in great details, on the many offshoots of the analysis methods, namely the solution of inverse problems and active control of waves. The thesis ends with brief summary of the tasks accomplished, significant contribution made out to the literature and the future applications that can be achieved from the proposed methods addressed in this thesis.